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On Size-Biased Logarithmic Series Distribution and Its Applications

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Abstract: In this paper, a size-biased logarithmic series distribution (SBLSD), a particular case of the weighted logarithmic series distribution, taking the weights as the variate values is defined. The moments and recurrence relation of (SBLSD) are obtained. Negative moments and inverse ascending factorial moments of the size-biased logarithmic series distribution have been derived in terms of hyper-geometric function. Recurrence relations for these moments have also been derived using properties of hyper-geometric functions. Different estimation methods for the parameter of the model are discussed. R- Software has been used for making a comparison among the three different estimation methods and with the logarithmic series distribution.

Key Words: Size-biased logarithmic series distribution, Negative moments, Inverse ascending factorial moments, Bayes' estimator, Beta distribution, R-Software.

1. INTRODUCTION

The logarithmic series distribution (LSD) characterized by a parameter α is given by

$$P(X=x) = -\frac{1}{\log(1-\alpha)}\frac{\alpha^x}{x} \text{ ; x= 1, 2…..} \quad (1)$$

The model (1) is a limiting form of zero-truncated negative binomial distribution. Negative moments of discrete distributions, mainly the binomial, Poisson and negative binomial have been investigated by various authors [Stephan [1], Grab and Savage [2], Mendenhall and Lehman [3], Govindarajulu [4,5], Tiku [6], Stancu [7], Chao and Strawderman [8], Gupta [9,10], Cressie *et al.* [11], Cressie and Borkent [12] and Roohi[13]. Inverse ascending factorial moments have only been dealt with by Lepage [14] and Jones [15]. Best *et al.* [16] discussed the test of fit for the model (1). Sadinle [17] linked the negative binomial distribution with the logarithmic series and Shanumugam [18] studied the characterization of model (1). A brief list of authors and their works can be seen in Johnson, Kotz and Kemp [19].

The first four moments of LSD are given as

$$\mu_1' = \theta(1-\alpha)^{-1}\alpha \text{, where} \quad (2)$$

$$\mu_2' = \theta(1-\alpha)^{-3}\alpha(1-\alpha) \quad (3)$$

$$\mu_3' = \theta(1-\alpha)^{-5}\alpha(1-\alpha)(1-\alpha^2) \quad (4)$$

$$\mu_4' = \theta(1-\alpha)^{-7}\alpha(1-\alpha)\left(\alpha^4+2\alpha^3-6\alpha^2+2\alpha+1\right) \quad (5)$$

The variance of LSD is given as

$$\mu_2 = \theta\left(1-\alpha\right)^{-3}\alpha[1-2\alpha+\alpha^2] \quad (6)$$

In this paper, we have made an attempt to study proposed size-biased LSD, its moments and recurrence relations. Negative moments and inverse ascending factorial moments of the size-biased logarithmic series distribution have been derived in terms of hyper-geometric function. Recurrence relations for these moments have also been derived using properties of hyper-geometric functions. In order to make a comparative analysis among the three estimation methods for the parameter of the size-biased logarithmic series distribution (SBLSD), one of the standard software packages R-Software is used which is meant for data analysis and graphics.

2. SIZE-BIASED LOGARITHMIC SERIES DISTRIBUTION (SBLSD)

A size-biased logarithmic distribution (SBLSD) is obtained by taking the weight of the LSD (1) as x.

We have from (1) and (2)

$$\sum_{x=1}^{\infty} x\cdot P(X=x) = \frac{\alpha\theta}{1-\alpha}, \qquad \theta = -\frac{1}{\log(1-\alpha)}$$

$$\sum_{x=1}^{\infty}\theta\ \alpha^x = \frac{\alpha\theta}{1-\alpha}$$

This gives the size-biased logarithmic series distribution (SBLSD) as

$$P_1[X=x] = \{{}^2F_1[1,A;A;\alpha]\}^{-1}\alpha^{x-1} \text{ ; x=1, 2…} \quad (7)$$

Where $0<\alpha<1$ and $\{{}^2F_1[1,A;A;\alpha]\}^{-1} = (1-\alpha)$

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